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Becker's evaluation of the achievements of Ancient Greek philosophy and mathematics

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ABSTRACT

Oskar Becker (an heir to Edmund Husserl and Martin Heidegger) showed a high appreciation of the Ancient Greek discoveries in mathematics. This appreciation was, at the same time, explicitly and clearly systematized. The systematization is concerned with the development of a couple of essential Ancient Greek mathematical theses, which include: Pythagorean theory of numbers; Zeno's discovery of the infinity problem, the discovery of the irrational and the question of the analytical experiment; Plato's theory of elements; Aristotle's theory of infinity and the abstraction of mathematical knowledge; and finally Kant's grounding of the limits of mathematical knowledge with regard to Aristotle.

Key words: number, infinity, irrationality, philosophy, mathematics, ἀρχή.

Oskar Becker (1889-1964) studied mathematics in Leipzig and Oxford and was qualified as a professor under Husserl's mentorship with the topic ***Die Stellung des Ästhetichen in Geistesleben*** (*The Position of the Esthetic in Spiritual Life*). His most important works include: ***Mathematische Existenz. Untersuchungen zur Logik und Ontologie mathematischer Phänomene*** (*Mathematical Existence. Studies on the Logic and Ontology of the Mathematical Phenomena*) from 1927, ***Das mathematische Denken der Antike*** (*Mathematical Thought in Antiquity*) dating back to 1957 and ***Grundlagen der Mathematik in geschichtlicher Entwicklung*** (*The Basics of Mathematics in the Historical Development*) from 1964. Beside these, his famous work ***Dasein und Dawesen*** (*Dasein and Dawesen*) can be found in the collection ***Zur Geschichte der griechischen Mathematik*** (*On the History of the Greek Mathematics*) from 1965.

The primary concern of this research was neither to establish the origin of the Ancient Greek mathematics nor to reconstruct its historical roots in Ancient Egypt or Babylon, but to analyze this science as a free one, i.e. a science that has its purpose in itself and not in its pragmatic or practical utility. Having probably been influenced by Heidegger, Becker sees Anaximander as the founder of that philosophical view of the world which originated from the medium of thought and not from the religious-mythical presentations. He considers Anaximander the philosopher who determined this view in terms of numbers, too. Thereby, the beginning of the history of the Western philosophy is associated not with Thales, but with Anaximander. A key reason to this, among others, may be the fact that there is a more substantial doxographic evidence concerning Anaximander's works in which some of the Pythagorean theses were anticipated. Anaximander's theoretical insight into the nature of macrocosm includes two problems: the number of macrocosms and the question of its inner structure, i.e. proportional relations.[1]

Still, the focus of Becker's analyses is the Pythagorean theory of numbers and its 'background' support in terms of its philosophical grounding, that is the Pythagorean arithmetic and geometry are the primary goal in comparison to the secondary status of the teaching of harmony. The sources Becker refers to are: Aristotle (***Metaphysics***), Euclid (***Elements***) as well as 'probably authentic' fragments by Archytas and Philolaus found in the Diels-Kranz collection (***The Fragments of the***

[1] Pavlović (1997), 67.

Pre-Socratic). Nevertheless, it is quite clear that Becker is primarily interested in the philosophical aspects of Pythagoreanism or, to be more precise, early Pythagorean interest in number and proportion. "For at one point the numbers are said to be the things themselves, at another to be within things, whereas, according to the third claim things are composed of numbers. Aristotle does not seem to make crucial differences among these claims."[2] Drawing support from Aristotle's interpretation of the Pythagorean theory of numbers, Becker is perplexed when it comes to the ontological status of numbers and wonders whether the numbers are essential beings (οὐσία) or the inherent basis of beings (ἀρχή). If this question is to be answered, one has to bear in mind that in his derivation of totality Anaximander did not take one component (element) in terms of matter (ὕλη) as a starting point, but the principle (ἀρχή) of the boundless (ἄπειρον). It is hard to imagine 'boundless' as an individual being of the material origin. However, it is possible to see it as a unity of 'boundless', i.e. the unity of the multiplicity of oppositions (contraries). Aristotle is determined in his claim that something unlimited and undifferentiated can be One (τὸ ἓν, principle) without gender or species distinctions.[3] This primarily seems to be concerned with Anaximander's "insight into the unity behind the phenomenal multiplicity, that is, the thesis about the unity of the foundation and an undetermined unity of the whole."[4] The result of Aristotle's analysis is that *ἄπειρον* can be neither an element (στοιχεῖον, elementum), nor the essence (οὐσία), but that it can be the principle (ἀρχή) in the sense of 'the oneness of origin', e.g. from water, air, boundless, etc. Things seem to be different with the Pythagoreans when the determination of the essence (οὐσία) of the whole being is in question. Following Aristotle, Becker is cautious in his formulation holding that the properties of numbers and things are somewhat similar "... especially in the structure of the musical harmony and in the composition of the firmament including its movement".[5] The relationship between things and numbers devised by the Pythagoreans,

[2] Becker (1998), 9.

[3] Aristotle, *Metaphysics* 1066b34.

[4] Žunjić (1988), 26.

[5] Becker (1998), 9.

and in Aristotle's interpretation, takes as its starting point the claim that the Pythagoreans considered numbers 'the essence of everything' (in existence), thereby the numbers are an essential part of the existing things, up to the point of, one could say, their being mutually identified. A passage from Aristotle's ***Metaphysics*** testifies about this claim[i]: "The Pythagoreans, on the other hand, observing that many attributes of numbers apply to sensible bodies, assumed that real things are numbers; not that numbers exist separately, but that real things are composed of numbers." (*Metaphysics* 1090a 20-25).[6] In the Pythagorean interpretation, according to Aristotle, numbers are not abstracted (separated) from the things, but are found in the sensible things as their essential parts (elements). Becker draws parallels between the (Pythagorean) claim that *οὐσία* of things consists in a numerical representation and Plato's claim that things participate in ideas, i.e. Plato's separation of ideas from things and Pythagorean non-separation of numbers from things. Therefore, numbers are not ideal, but real entities that play the role of the principles (ἀρχαί) and elements (στοιχεῖα).

Becker thinks that the description of the Pythagorean theory of numbers faces certain contradictions and paradoxes. In order to determine the 'nature' of numbers he formulated one such paradox: "... if the numbers are in things or if the things are composed of numbers, then things are not simply numbers".[7] Accordingly, Becker observes the paradox in the hypothetical formulation we have encountered in Aristotle's text, and that is 'things are numbers', so he suggests a change that should infer "the immanence of numbers in things, regardless of their being understood as either the composite parts, on one hand, or as elements (στοιχεῖα) of numbers being identified with the elements of things, on the other".[8] Regardless of whether the first or the second inference is the right one, those elements are the boundary (πέρας) and boundless (ἄπειρον). The immanence of numbers in things is considered by Becker the 'arithmetic' structure of things. It is therefore hard to deny that the numbers are the essence (οὐσία) of things. However, the origin of the Pythagorean theory of numbers is still at stake. An opinion that

[6] Žunjić (1988), 35.

[7] Becker (1998), 10.

[8] Ibid.

has been entertained in an extraordinary study ***Prostor vreme Zenon*** (*Space Time Zeno*) by Miloš Arsenijević (drawing support from the rich doxographic evidence) is that the primary source of the Pythagorean theory of numbers "... was the discovery of a fixed relation in the length of the cords through which the musical fourth, fifth and octave were produced, that is the musical intervals that the Greeks considered consonant, harmonious".[9] Aristotle's definition of accord (harmony) provides textual support for the Pythagorean teaching of numbers. This harmony is also defined in terms of the numerical relation between high and low pitch. Becker's interpretation corresponds to this line of determining the sources (cf. footnote 5). On the other hand, he has a correct insight when he claims that Aristotle's testifying about the relationship between things and numbers is supported by several different theses: things are numbers, things exist through the imitation of numbers, things are elements of numbers – elements of everything in existence, numbers are the causes of the essence. On the basis of the analysis of the Pythagorean teaching of numerical intervals it can be inferred that the Pythagoreans made a generalization of its own kind with respect to all things: "everything is ratio and number (λόγος καὶ ἀριθμός), and this is perhaps the whole point of the original Pythagorean teaching of numbers".[10] We are also informed by Aristotle that the Pythagoreans were the first to start the conversion of numbers into geometrical shapes (triangle and square), thereby, they established their cosmology that is based on "the contemplation of shapes" as fore-forms (of the geometrical and numerical series).[11] The difference clearly observed by Becker is the one between an ideal character of numbers (Plato and the Pythagoreans) and the Heraclitean determination of sensibly given current things. Plato's theory of (ideas) numbers establishes a mathematically founded set of relations among the most general concepts, thereby reducing those most general concepts to an ideal series of numbers.

[9] Arsenijević (1986), 46. According to Arsenijević, the numerical relations are essential for musical intervals: the musical fourth (4:3), fifth (3:2), octave (2:1).

[10] Arsenijević (1986), 47.

[11] Aristotle, *Metaphysics* 1092b.

Namely, it is possible to numerically determine all ideas due to their connection (relation) to numbers, but only some ideas are the ideas of numbers (numbers-ideals or ideal numbers). Following the analogy ideal-sensible concerning the issue of numbers, Becker, having Plato in his mind, strictly separates 'arithmetic one' from the concept of 'the counted number', as well as the latter from 'ideal number' or 'idea number'.[12] Accordingly, ideal numbers have a special status within so called 'most general concepts' (universals) due to their being generated from the first principles (One and indeterminate Dyad); (Ger. Das Eine und die unbestimmte, unbegrenztze Zweiheit).

Aristotle also differentiated between 'the counted number' (the one that can be observed on the sensibly given things) from 'the number with which we count', which would, in fact, be a monadic number, i.e. the principle of numbers or One (μονάς). Becker's establishment of the link between the Pythagoreans and Plato is justified. The Pythagoreans converted geometry into arithmetic, whereas Plato converted arithmetic into geometry, which can be clearly observed in the dialogues such as ***Theaetetus*** and ***Timaeus***. An example of the Pythagorean conversion of geometry into arithmetic is the famous tetraxis decade whose graphic representation resembles an equilateral triangle drawn using ten points (in 4 rows). The sum of the points in four rows (1+2+3+4) equals 10, and 10 is the number consisting of an equal number of odd (1, 2, 3, 5, 7) and even ones (4, 6, 8, 9, 10).[13] Another example that is illustrative for Plato's geometrization of arithmetic (unwritten teachings, ***Timaeus***) is the one in which the elements of the physical world are stereometrically represented as regular polyhedrons: "(fire, water, air, earth : tetrahedron, icosahedron, octahedron, cube) are dimensionally/analytically reduced via planes and directions to pointed monads (indivisible directions) as the last units of extension they themselves present the derivative units – alongside hyletic addition in multiplicity".[14] On the other hand, we are reminded by Becker that the smallest units of the components of the existing things in Plato's ***Timaeus,*** i.e. so called elements are geometrically represented, in which case fire corresponds to square, air

[12] Becker (1998), 11.

[13] Pavlović (1997), 109.

[14] Krämer (1997), 128.

to octagon, water to icosagon and finally in this representation the earth corresponds to cube. "The first three solids whose planes are the congruent equilateral triangles have been envisaged as consisting of three planes – therefore, elementary three-dimensional bodies have been envisaged as being composed of the pure two-dimensional planes." [15] Aristotle was against such a construction because for him the planes were abstract mathematical creations and because he believed that the entities of the higher dimensions could not be composed of the lower level ones. This means that the lines do not consist of points, planes do not consist of lines and bodies do not consist of planes. Although the concept of matter did not exist in the Ancient Greek language, the term *ὕλη* (approximately 'matter') referred to something else, most precisely speaking the ultimate cause of the phenomena was not considered of hyletic, but of mathematical origin referring to symmetry, shape or the mathematical law. Becker himself is hesitant in claiming that the sense of Plato's term *ὕλη* could be rendered into contemporary terms such as 'space, matter and field'. [16] For Plato's basic triangles to be precisely mathematical, analogical deduction that includes the construction of 'the cosmic soul' is required, which is clearly a non-hyletic scheme or a model of the visible universe. "It is true that Plato speaks of the eternal kingdom of space (chōrā, *Timaeus* 52ab). However, the question is whether this property of eternity should be considered in a necessary relation with mathematical necessity. This question would require a detailed study; at any rate there is an impression of Plato's conspicuous pythagorisation precisely in his elementary teaching."[17]

In ***Physics*** Aristotle comments on Plato’s identification of matter (ὕλη) and space, i.e. the identification of place and space. He also adds that ideas and numbers are not in space if the space is included in participation either as a secondary principle (indeterminate Dyad) or as *ὕλη*.[18] Mentioning the secondary principle, the indeterminate Dyad (the first principle being One) is very important for us since it is from the secondary principle that places, vacuum and that which is infinite are

[15] Becker (1998), 13.

[16] Ibid.

[17] Becker (1998), 15.

[18] Aristotle, *Physics* 209 b 11-17.

derived. It is precisely this assumption that is relevant for the mathematical construction of the universe. "Plato himself, even though he reduces everything to the principles, seems to be concerned with other issues as well connecting them to ideas, ideas with numbers, and numbers with the principles."[19]

Nevertheless, in ***Metaphysics*** Plato's theory of the principles (from his unwritten teachings) is interpreted by Aristotle as ideas being considered the causes of all beings and the elements of the ideas simultaneously being the elements of everything else. Accordingly, in terms of *ὕλη*, the principles are great-and-small, whereas in terms of *οὐσία* the principle is One "… for ideas-numbers originate from great-and-small according to their participation in One".[20]

Similar to the Pythagoreans Plato considers ideas (numbers) the causes of everything in existence. What is original in his discovery is that instead of One as boundless he proposed the indeterminate Dyad, whereas the boundless itself is composed of great-and-small. Another difference lies in the fact that Plato separated numbers from sensibility unlike the Pythagoreans who regarded numbers either as things themselves or being within them. There is also doxographic evidence in favour of the similarity between the Pythagorean theory of numbers and Plato's theory of the principles. For example, in his ***Commentaries on Aristotle's Metaphysics*** Alexander of Aphrodisias claims that the Pythagoreans and Plato proposed numbers as that which is the first and most simple. Accordingly, since 'ones' are numbers, numbers are the first among beings. The principles of numbers are also the principles of ideas or, to put it differently, the principles of ideas as numbers are the principles of numbers and these include One and indeterminate Dyad (μονάς και ἀόριστος δυάς). The principle 'indeterminate Dyad' generates great-and-small and it is boundless and infinite. [21] Great-and-small belongs to the nature of the boundless. "Namely, there where they are present and where they contribute to strengthening or weakening, that which participates in them does not stop or end, but continues into

[19] Krämer (1997), 337. *Testimonia Platonica* (quotation taken from Theophrastus' *Metaphysics* 6a15-b17).

[20] Krämer (1997), 339. Aristotle, *Metaphysics* 987b20.

[21] Krämer (1997), 343.

infinity of indeterminedness."[22] In his work ***Against the Mathematicians*** Sextus Empiricus writes that they (the Pythagoreans and Plato) defined the point by analogy with the concept of one because the point is a kind of a beginning of a line, whereas one is a kind of a beginning of other numbers: "Thus, just as a point contains a relation to Oneness, a line is regarded in terms of the idea of indeterminate Dyad. Both are envisaged through some kind of transition. In other words, the line is the length envisaged between two points and (is) void of breadth".[23] A point, therefore, corresponds to one, a line to two, a plane to three, a body to four. There is also an assumption according to which a body is composed of (or more precisely initiated by) the point in movement which amounts to the line, the line in movement comprising the plane, whereas the plane positioned in the height produces the body with three extensions: the length, breadth and height. In his ***Commentary on Aristotle's Physics*** Simplicius quotes Plato's follower Hermodorus: "… in that which is thought of as great-in-comparison-to-small everything contains that which is more-and-less: accordingly, in that which is more-or-less the 'higher' is possible due to its continuation into infinity; the same holds for broader-narrower, lighter-heavier as well as everything else that is predicated in that way will be thought of in terms of infinity."[24] The question that would be posed from Pythagoreans to Academy was a not entirely resolved relationship between One and multiplicity that originates through generative and derivative processes of being. In a more radical (polemical) form this question could be formulated in the following way: "Does One generate multiplicity as the principle of differentiation between numbers and things, or is it itself generated through the impact of the first principle of difference (boundary) on the multiplicity (boundless)?" [25] It is easy to accept the first solution offered, which means that this generation (origination, derivation) is to be understood physically, and not, among other ways, in terms of numbers.

[22] Krämer (1997), 345.

[23] Krämer (1997), 359.

[24] Krämer (1997), 363.

[25] Žunjić (1988), 40.

The logical, methodological and, generally speaking, heuristic direction of Becker's study of the Pythagorean teaching of number and proportion leads to its connection with Plato's theory of principles (unwritten teachings) and his late philosophy. Another line of reasoning connects the Pythagoreans to the Eleatic philosophy "... which appears to have originally been tightly connected to Pythagoreanism (Parmenides' teacher was allegedly a Pythagorean Ameinias) and which would have to resolve the generation problem from within One as well as the relationship between One and multiplicity." [26] In comparison to the historiography and chronological study of Ancient Greek philosophical-mathematical teachings, Becker (as well as we) is far more interested in the influence and systematic links between the Pythagorean teaching and Eleatic and Plato's philosophy. At this point we can affirm that the Pythagoreans, Eleatics, Plato and Aristotle provide a foundation of the Ancient Greek philosophy of mathematics. The concept of knowledge and the research method could be termed dialectically-totalizing, for a 'dialectician' is the one who can observe one form beyond multiplicity, i.e. the one who has an insight into general concepts (ideas) which individually represent oneness, sameness and substantiality. Semantically speaking, 'idea' has a double meaning: a conceptual meaning of generality and an ontological one referring to formality. "Idea is the synthesis of these two aspects which correspond to two alternative terms used by Plato: *εἶδος* and *ἰδέα*. These terms are derived from the same root, however, they do not share the same meaning. The term *ἰδέα* is derived from the verb *ἰδεῖν*, whose present tense results in 'to see', whereas its past tense gives rise to 'to know' (οἶδα). This is not an accidental transition: what is known is what has been seen, the appearance and form ... The term *εἶδος*, therefore, refers to the form of that which is known." [27]

Becker dissociates himself from an idea that Plato could be understood as a Pythagorean. For him Plato is primarily a critical thinker who clearly emphasized the concept of a model in mathematical exact science, since what serves as a model in modern physics can be traced back to Plato's late philosophy in which a geometrical

[26] Ibid., 41.

[27] Ibid., 96.

demonstration of a form (model) is used in order to move through the theory. "Using a determined set of phenomena a modern physicist creates 'a model' and studies it, so to say *in abstracto,* observing which properties of the perceived phenomena have been reflected in it – how much within phenomena it 'preserves', which is what the antique formula proposed." [28]

A key critical note of Aristotle’s concerning the (Academicians') theory of ideas has been given in ***Metaphysics***[ii]: "... for whereas the Platonists derive multiplicity from matter although their Form generates only once, it is obvious that only one table can be made from one piece of timber, and yet he who imposes the form upon it, although he is but one, can make many tables." [29] The point of this criticism is that Platonists do not understand that *ὕλη* is in fact one, whereas *εἶδος* as a form is multiple, i.e. it multiplies through the one who 'keeps in his eye' (carries/imposes) that form (in Aristotle's case the carpenter). The link between generality and formality is defined by Aristotle via the distinction between a physical form and noetic concept, via the differentiation between oneness of species and number, respectively. "Physical forms (shapes) are one in terms of species, but not in terms of number. A noetic form is numerically one, but, precisely because of that, it does not make much sense to say that is one in terms of species."[30] The existence of forms is seen by Aristotle only in hyletic forms, not separately from them in a world of ideas.

Plato's polemics with the Eleatics in his dialogue ***Parmenides*** aims at demonstrating not only that the being is not one, but also that there is not just one being and nothing else. The consequences of making One absolute (i.e. one being as identification of all and One) are insupportable. "The Eleatic thesis on being conceptualized as one and only totality (ὅλον) is proven self-contradictory. In order to describe their one (and only) being, the Eleatics use the multitude of names, thereby principally damaging the oneness they want to protect…".[31] For Plato, introducing ideas created a possibility of organizing sensible

[28] Becker (1998), 17.

[29] Aristotle, *Metaphysics* 988a2-4.

[30] Žunjić (1988), 100.

[31] Ibid., 109.

multiplicity, the aim was not 'to freeze' the totality of being. With regard to that, Zeno had already warned against ascribing different predicates to one and the same thing, which procedure brings about it not being one, but multiple. Dialectical considerations of the ontological status of One 'when it is' and One 'when it is not' resulted in Plato's dissociation from the Eleatic being and his approximation to One which is at the basis of everything in existence.[32]

Using dialectics as a method of the logical analysis of concepts, Plato established two conceptual patterns whose prevailing relationship is complementary. However, this relationship is occasionally competitive: "elementalistic type by analogy with a mathematical model through which everything is reduced to the ultimate, most simple elements; this type is especially concerned with reduction via division of numerical and dimensional series into progressively simpler elementary parts; b) generalistic type…which ascends from the individual to a progressively more general level …"[33] The generalistic type is primarily related to the field of most general concepts, i.e. the field of meta-ideas which comprises the functional aspects of One (the same, similar and equal) and their contrary and contradictory oppositions (the other, different, non-equal). These reflective concepts perform a regulative function in terms of ideal numbers. Mathematics with its axiomatic system, and the unity of three fields of study (arithmetic, geometry and astronomy) belongs to an order of knowledge within Plato's unwritten teachings, whereas individual fields of being are determined through special principles: "monads (for numbers), the points, i.e. indivisible direction (for all forms of movement), the temporal unit of moment (νῦν for time), etc."[34]

All of the special principles listed (monads, indivisible direction, circular movement, temporal unit) are dependent on the precisely determined mathematical proportional relationships (λόγοι, ἀναλογίαι) and "... according to ontologically understood division and addition (ἀφαίρεσις – πρόσθεσις), according to the relationship between priority and posteriority (πρότερον – ὕστερον) as well as according to the

[32] Ibid., 112.

[33] Krämer (1997), 130.

[34] Krämer (1997), 134.

relationship of non-discontinuity." [35] For instance, the arithmetic 'one' is a point without extension. However, in opposition to that, a point as an indivisible direction is a 'one' with extension: "Therefore, in a progression from arithmetic to geometry, number is ascribed extension as an index of all geometrical essential features (πρόσθεσις) or, contrary to that, extension diminishes if we go back from geometry to arithmetic (ἀφαίρεσις)".[36]

Becker reminds us that in the Hindu and East Asian mathematics there is no evidence of an explicit application of infinity, not even in the form of total induction (inference from n to n+1). While studying Ancient Greek philosophies, Becker found evidence of infinity emerging in the form of a convergence of unlimited series, and within the proofs against movement (cinematic paradoxes) at that, which had been devised by Zeno of Elea in the paradox named ***Dichotomy***. ***Dichotomy*** demonstrates that it is impossible to cover a distance from point A to point B of a certain trajectory in a determined limited time, since it is impossible to surmount the unlimited number of points on that path. "The manner of demonstration as well as the terminology used by Aristotle and Commentators make it possible for the 'surmounting' to be understood as ...reaching the points and crossing the parts of the path".[37] Surmounting infinity can be understood as reaching infinitely many points, on one hand, or as crossing an infinite number of parts of a path, on the other. Beside the respective formulations of reaching and crossing, Arsenijević (following Aristotle) points to the analogy between crossing the parts of a path and performing an infinite series of

[35] Ibid., 135.

[36] Ibid. Perhaps we can use this occasion to explain the inscription at the entrance of Plato's Academy: "Let no one who is not a geometer enter". Namely, it is clear that if the level of sensible objects in movement is taken to be the starting point in the hierarchy of the whole being, one uses the abstraction (separation) procedure in order to move towards (intelligible) mathematical entities and thus reach the theory of ideas, and subsequently the theory of principles as well. The knowledge of geometry is, therefore, a necessary prerequisite for one to reach the theory of ideas, and then the theory of principles.

[37] Arsenijević (1986), 80.

successive acts, that is the analogy with the counting of an infinite set.[38] To put it picturesquely, one cannot touch each of infinitely many points that are found within a certain (limited) part. Aristotle's commentators, doxographs Philoponus and Simplicius present ***Dichotomy*** according to the decreasing geometrical progression: $1/2 + 1/4 + 1/8 + 1/16 + ... + 1/2^n = 1$.

$$\underline{1/2 + 1/4 + 1/8 + 1/16 + ... + 1/2^n} = \sum_{n=1}^{\infty} (1/2)^n$$

This means that for one body to cross the whole line, it must cross a half of its length beforehand, one quarter of it before the latter, one eighth, one sixteenth before the previous lengths and move accordingly *in infinitum*. However, ***Dichotomy*** can be described in terms of the increasing geometrical progression as well: $\underline{1/2 + 3/4 + 7/8 + \ldots + (2^n-1)/2^n}$. (n=1, 2, 3,…) The third option is to use a part of the line:

A C C′ C′′ B, AC+CC′+CC′′+ ... in infinitum = AB.[39]

According to Becker, the paradox of the converging infinite series is contained in the fact that, on one hand, in the decreasing geometrical progression a series (1/2 + 1/4 + 1/8 + 1/16 + ...) increases, whereas, on the other hand, the series does not increase into infinity since its sum is always less than 1 "... no matter how long the series continues".[40] In fact, there is no logical contradiction found in this 'on one hand and on the other hand' formulation. This is because something can continually increase without crossing all the limits. The only condition that needs to be fulfilled is for that increase to parallel a sufficiently rapid decrease in the course of time. Becker considers a qualitative assessment the characteristic of the Ancient Greek way of thinking. The qualitative assessment presumes that if something continually increases it has to cross all the limits. In comparison to qualitative, a subtle quantitative thinking is far less represented.

The difference between the construction with a decreasing and increasing geometrical progression is not entirely without significance. If the increasing geometrical progression (in Aristotle's interpretation) is

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[38] Ibid., 81.

[39] Becker (1998), 82.

[40] Ibid.

assumed, there is a close connection to Zeno's proofs against multiplicity: "... he let the racer move and proved him incapable of reaching the goal, so that he could, conceivably so, infer that there can be no movement at all since there is no reachable goal." [41] As for the construction with a decreasing geometrical progression (commentators Simplicius and Philoponus), the racer is in fact prevented from starting the movement (he remains motionless) because we have previously determined that he cannot reach the finishing point as he is incapable of surmounting infinity successively (step by step) and that this holds for every goal.

The reason for the convergence of infinite series is not seen by Becker in the unlimited diminishing of the members (halved in comparison to the previous magnitude) that, thereby, become smaller than any magnitude we could introduce: "That it is not so is demonstrated through the so called harmonious series:

$$1/2 + \underbrace{1/3 + 1/4}_{>1/2} + \underbrace{1/5 + 1/6 + 1/7}_{>1/2} + 1/8 + \ldots$$

Unlimited series increase can be observed."[42]

Zeno's proofs against movement seem to be more operative in comparison to the proofs against multiplicity. The reason for this is that in the so called cinematic paradoxes it is enough to let the division continue without limits (dynamic infinity), and then using the *reductio ad absurdum* model the opposite hypothesis can be refuted (namely, that the infinity can be surmounted step by step). In a word, accepting movement results in accepting mutually accepted propositions, whereas, if the logical principle of non-contradiction is accepted, it is clear that this cannot be allowed. Thus, the result of Zeno's negative dialectics is that there is no movement and there is no multiplicity. Still, it is one thing to accept that there is no movement and quite another that it is impossible, and "… it is difficult to imagine a philosopher giving up the search for a further explanation once he is halted at the construction that the theory he accepts proposes that which he otherwise believes is impossible."[43]

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[41] Arsenijević (1986), 82.

[42] Becker (1998), 70.

[43] Arsenijević (1986), 98.

Both ***Dichotomy*** and ***Achilles*** are separated by Becker from some kinds of finitistic considerations since these paradoxes represent an infinite process and a division of a finite length into the parts whose number unlimitedly increases. "Individual lengths that are reached in the course of the infinite process of dichotomy can certainly be represented in perception through an image, but this does not hold for the whole process."[44] In terms of interpretation, the most important thing for Becker is to emphasize the Ancient Greek discovery of infinity and its significance for philosophy and mathematics. He finds the highest point of the consideration on the nature of infinity and continuum in Aristotle's ***Physics*** (III 4-8) whose continuing relevance has been ensured up to our time.

According to Aristotle, that which is unlimited exists only in potentiality. However, the given hyletic infinity does not resemble the potentiality of a particular matter (e.g. the marble of a statue to be), but is "... like the potentiality of the days or Olympic games" that are repeated all over again.[45] Still, Aristotle uses the actual concept of infinity (*Physics* 206b23-24). Infinity is shown in the processes of addition and division and the dichotomy scheme (Zeno) can be revised in a way that turns the division ratio 1:1 into that of 1:2 or 3:2. Aristotle explicitly rejects the possibility of completing the division. Any magnitude can be divided, however, this division can neither be completed nor performed until the end. "Accordingly, the continuum is divisible (only) in terms of the further divisible parts." [46] To say that a particular magnitude is potentially infinite means only undeterminedness with regard to its possible parts (*in indefinitum*), thereby, the potential division is indefinitely far away.

According to Arsenijević, Aristotle's indefinitism can be accounted for either through the two types of infinity or the two types of potentiality. Infinity in categorematic terms is impossible, whereas it is possible in syncategorematic terms (as the absence of an upper limit in a certain process). "In the first case, we can differentiate between infinity as an unlimited number of things in existence and infinity whose

[44] Becker (1998), 72.

[45] Ibid., 80.

[46] Aristotle, *Physics* 231b16.

number of things originating or being capable of origination is not fixed. In the second case, we can differentiate between the possibility of existence and the possibility of origination."[47] In ***Categories (Organon)*** continuity is divided into the discrete (number, speech) and continuous one (line-plane-body, time and place). Aristotle takes the straight line as the paradigm of continuous quantity because there is no interruption at a single point. The line is the boundary of the plane, whereas the plane is the boundary of the body. Time is also a continuous (uninterrupted) magnitude because the present time (moment) represents a common boundary of the past and future. For Aristotle, a magnitude is continuous if and only if it is not discrete.[48] His objective was to separate continuous magnitudes from those whose mathematical model is, for instance, a set N (a set of natural numbers). He clearly states that the numbers are not continuous, but discrete quantities because by numbers he means the natural ones (the ones we use for counting). In the Ancient Greek mathematics there was no clear distinction between the rational and real numbers. This was because the real numbers that are not rational (i.e. that are irrational, e.g. $\sqrt{2}$), were not even regarded as numbers, but figured as a geometrical relation of incommensurability.[49]

Alongside Becker, we can regard the relationship between the number and line as the one between the discrete and continuous quantity. In that context, it is not only that Aristotle claims that a continuous magnitude, such as a body, is one, whereas it is potentially multiplied in the sense of the infinity of the possible parts. He also claims that "... it is impossible to state in any sense that a particular whole consists of infinitely many parts." [50]

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[47] Arsenijević (1986), 166.

[48] Ibid., 167.

[49] Becker questioned the hypothesis that the first thing discovered was the incommensurability of the diagonal and a line of the square, and this means irrationality -$\sqrt{2}$, after Fritz's (Kurt von Fritz) studies of the Pythagoreans which suggested that the first thing to have been discovered was the incommensurability of the diagonal of the regular pentagon with its line "… and that it was discovered by Hippasus, or at least was discovered in his time". Becker (1998), 74.

[50] Arsenijević (1986), 166. This is why Arsenijević is right when he claims that Aristotle is an anti-infinitist in a strong finitistic sense.

The point, line and surface as boundaries and geometrical shapes do not exist separately from the specified geometrical shapes. From the logical perspective, the point has a priority over the line, whereas the line is prior to a triangle. This definition of the mathematical line will be subsequently rejected by Aristotle. Thus, he will consider the point not the boundary, but the constituent of the line. A limited line (a closed line segment) has its beginning and end in points. A logical priority (precedence) does not imply an ontological priority (of substance). Accordingly, Aristotle, in a somewhat unusual way, reaches a conclusion that the physical bodies have an ontological priority in comparison to mathematical objects.[51] The explanation for this consists in the assumption that, with regard to their existence, mathematical substances have an allegedly lower 'degree' of substantiality than the bodies we perceive (physical substances). "And if we are allowed to speak of geometrical substances, they exist as external shapes and the boundaries of the physical ones …"[52] It cannot be said that a particular physical substance (a perceived body) consists of points, lines and surfaces. What can be talked about is that there are potential points, lines and planes with regard to potential division through which they would be actualized. The number of the potential points, lines and planes in that division would be indefinite, but not also infinite in the categorematic sense. Crossing a part of a particular path successively (step by step) does not mean that we have surmounted infinity. It means we have surmounted definitely many actual and indefinitely many potential sub-distances. Providing the racer moves continually and at an invariable speed, he will reach the end of the stadium in a limited time, that is, he will surmount a limited number of the real parts of the path and an indefinitely big number of the potential parts of the path (but not also an infinite number of them).

The discovery of incommensurability and Zeno's aporiae led to the finitistic model of the Archimedes' axiom which, in the last analysis, tries to 'avoid' the appearance of infinity. The finitism of Archimedes' method

[51] Aristotle, *Metaphysics* 1077b.

[52] Arsenijević (1986), 168.

(procedure) "... is based on the hypothesis that the number of inscribed and circumscribed figures must be finite..."[53]

Archimedes' axiom (which claims that surmounting the distance in space from point A to B takes a limited number of steps) implies the so called inclusion principle according to which a certain point (or a certain real number) can be inserted on a certain line between two points (two rational numbers). We used Aristotle's writings mainly as a source of interpretation of Zeno's paradoxes. However, now is the time to consider Becker's evaluation of Aristotle's teaching on infinity and continuity. Principally speaking, what is relevant for Aristotle's understanding of the continuum is that, for example, a particular line segment does not consist of points, but that infinitely many numbers of points are potentially given in that line segment, so that they can be produced through division or some other mathematical operation of the constructive type. On the other hand, in Georg Cantor’s theory of sets it is claimed that the line segment is an actually infinite set of points which are separated through division for the purpose of consideration. In that sense, for Aristotle, continuous magnitudes are actually undivided, however, they are potentially divisible into infinity. The difference between an actual and potential part of a certain continuum consists in the fact that, through the transition from potentiality into actuality, the part in question would become something individual and autonomous by being separated from the continuum. Aristotle does not speak of the actual parts of the continuum, but he does speak of the potential ones, which means that the structure of the continuum can be spoken of in hypothetic terms, with a special note to the fact that we cannot count with the possibility for every part of a certain continuum to be actualized. Continuous magnitudes are divisible into parts that are always further divisible, because if something represents a part of a certain whole, Aristotle expects the part to represent all essential properties of that whole.

Both parts of Aristotle’s continuum understanding are equally important: firstly, that the continuous magnitudes represent something without parts, and secondly, "… that it is still permitted to talk about the structure of the continuum with regard to different relations in which its

[53] Arsenijević (1986), 202.

potential parts are found".[54] Unlike Plato, who positioned mathematical entities (as rational and intelligible) between sensibly given phenomena and *εἶδος*, Aristotle defined that which is mathematical (in knowledge) starting from the term 'ἀφαίρεσις' (subtraction), which was subsequently rendered into Latin 'abstractio' (drawing away). As Becker himself indicates, the term 'abstraction' is ambivalent in that it requires 'bracketing' of certain aspects or properties of a particular thing and, in addition, directing the focus to some other aspects. Besides, 'abstracting' must also include 'ascending' to more general concepts (that which is common, κοινός, i.e. the mathematically general of a species). "Accordingly, mathematical things, which are not in themselves separate, are thought of as separate."[55]

This is one of the main reasons why a mathematician is in 'opposition' to the philosophers of nature or metaphysicians. As a subject of his study a physician has a world of sensibly changeable entities, whereas a metaphysician studies the world of unchangeable substances. A mathematician, nevertheless, is yet to gain the sense of the mathematical through his spiritual activity via abstraction (via intentional object of consciousness). It is in this that Becker recognizes nominalistic moments of their own kind in which mathematical things emerge as mental entities (separated in thoughts, reflection), whereas they are not separate in themselves. The issue of 'ascending' to more general concepts, which we have set forth as one of the functions in abstracting, should not be understood as a generalization towards the gender of a higher level of generality, but as "... a formalization that is derived from all material regions and their highest genders and transcending them".[56] The tendency towards formalization of the movement towards higher genders is found by Becker in the character of the Ancient Greek mathematics which he illustrated by two types of examples: "Firstly, this involves certain propositions concerning equal and unequal, for example, the equal subtracted from the unequal amounts to the equal and similar examples. Secondly, it refers to the

[54] Arsenijević (2003), 27.

[55] Aristotle, *Metaphysics* 998a.

[56] Becker (1998), 84.

general teaching of proportions…"[57] Through the abstraction a mathematician observes different forms (points, lines, planes, bodies) with regard to quantity and continuity, but also with respect to that which is discrete (e.g. number). The main delineation criterion for differentiating between the Pre-Ancient Greek and Ancient Greek mathematics is seen by Becker in the explicit consideration of that which is infinite, which we observed through the example of Zeno's dichotomy (as an infinite geometrical series). Another great discovery refers to irrationality. The essence of infinity was seen by Aristotle in the process that cannot be unlimitedly continued or, ontologically speaking, infinity has its being in the form of potentiality. A distinctive feature of the mathematical way of thinking was seen by Aristotle in abstraction in which mathematical objects are observed as those that originated from concrete physical things through 'leaving them aside' (ἀφαίρεσις).

It was only a long time after Aristotle's study of infinity that a German mathematician Cantor refuted the thesis of infinity having its being in potentiality. The thesis was refuted through the rejection of Euclid's axiom according to which 'a whole is bigger that a part'. Summarizing Aristotle's theory of abstraction (of that which is mathematical) Becker concludes that it refers to "… that which is present alongside physical bodies, although they are idealized, thereby, that which is common to mathematical creations is also taken into consideration, for example, the proportions".[58] Beside Aristotle's indefinitism and Cantor's theory of sets it is also necessary to mention Kant's re-actualization of Aristotle's theory of the continuum. We should be reminded that Aristotle, being an anti-infinitist, refuted not only the real infinity, but also the infinity of the potential parts. However, he allowed the possibility of the continuum (e.g. a straight line) that could be divided into as many parts as desired, but finitely many. On the other hand, Kant rejected the possibility of the actualization of the unlimited number of potential parts. However, he allowed the infinity of the potential parts of the body. The difference between Aristotle's and

[57] Ibid.

[58] Becker (1998), 140.

Kant's indefinitism is classified as the difference between a physical and purely mathematical space.[59]

Kant himself notes that indefinitism is one of the possible *a priori* qualities of a certain quantity.[60] The difference between Aristotle's and Kant's indefinitism lies in Kant's claim that one cannot specify the biggest possible number of parts from which a certain body is composed, however, one can always state that it consists of finitely many parts. On the other hand, Aristotle does not allow an infinite number of the potential body parts. Considering the problem of the relationship between a part and a whole, Kant differentiated between two types of wholes (compositions): *compositum reale* and *compositum ideale* (real composition and ideal composition). The real whole is the one that is composed of the parts that precede it, whereas the ideal is the one whose parts can be disassembled, but which does not consist of parts. Any continuous magnitude (*quanta continua*), space and time for example, is an unlimitedly divisible magnitude. The real whole is preceded by real parts which should be understood not as points (moments) but as parts of an interval. "I call an extensive magnitude the one in which the representation of the parts enables the representation of the whole (thereby, it necessarily precedes it)."[61] However, at another point (the notes on the second antinomy thesis) in the ***Critique of Pure Reason*** Kant explicitly opts for *compositum ideale*: "Space (as well as time or any other magnitude for that matter) should not be in fact called a *compositum*, but *totum* since their parts are made possible only within a whole (based on a whole), and it is not the whole that is made possible through the parts. Indeed, space could be called *compositum ideale*, and not *compositum reale*."[62] The reason for Kant's using the term *totum* rather than *compositum ideale* lies in the fact that Kant actually regarded the ideal composition as a *pseudocompositum,* since it does not presuppose the parts as elements and, therefore, not being a real *compositum*. "The noun 'composition' (compositum) and the verb 'composed of' (componit) can be ascribed the respective adjectives 'real/ideal'

[59] Arsenijević (1986), 172.

[60] Kant (1976), A176.

[61] Kant, A162.

[62] Ibid., A438.

(reale/ideale; realia and idealia are the nominative plural of these adjectives) and adverbs 'real/ideal'. The ascription is based on the character, in fact on the modality of the parts, on their actuality, reality, that is, their potentiality, unreality, ideality."[63]

The only conclusion we can make based on Kant's hesitation concerning the formulation of *compositum ideale* or *totum substantiale phaenomenon* is that a continuum as a whole is composed of the potential, but not actual (real) parts. This means nothing but the fact that Kant gives priority to the whole of spacial-temporal continuum over its parts as intervals. It would not be an exaggeration to claim that Kant overtook and further developed Aristotle's indefinitist theory of the continuum (unlimited divisibility of magnitudes indefinitely far) in terms of interval composition. This theory is aimed against the theory of the *discretum* according to which there is no unlimited divisibility, neither the partial nor complete division of the unlimitedly divisible magnitude. Indefinitism includes an unlimited multitude of the possible (and in an indefinite distance) parts of a whole as well as a limited multitude of the real parts of a whole. The mathematicians, who predominantly study pure space (the space void of physical bodies), give priority to Kant's indefinitism over Aristotle's because they speak of space in the modality context.

Oscar Becker, a great German philosopher and mathematician, indicated that after the Pythagorean golden age of arithmetic (as logistics), the development of this science was directed towards the growth of geometry because the geometrical relations became determinable even when they could not be expressed through numbers.[64] Historically speaking, what happened was an 'emancipation' of arithmeticised geometry from arithmetic. "The same thing that had happened with arithmeticised geometry also happened with the modern mathematicised physics due to a natural tendency for the movement of

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[63] Jakovljević (2012), 70.

[64] Frege is right when he reminds us that number should nor *a priori* be associated with arithmetic and specifies Leibnitz's opinion according to which number is something general, that it is actually a mathematical figure. Frege (1995), 49-50.

physical bodies to be understood as the movement in a previously structured mathematical space."[65]

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[ii] Ibid.

[65] Arsenijević (1986), 409.